Averaging and Cosmological Observations

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The gravitational field equations on cosmological scales are obtained by averaging the Einstein field equations of general relativity. By assuming spatial homogeneity and isotropy on the largest scales, the local inhomogeneities affect the dynamics though correction (backreaction) terms, which can lead to behaviour qualitatively and quantitatively different from the Friedmann-Lemaître-Robertson-Walker models. The effects of averaging on cosmological observations are discussed. It is argued that, based on estimates from observational data, the backreaction (and, in particular, the averaged spatial curvature) can have a very significant dynamical effect on the evolution of the Universe and must be taken into account in observational cosmology.

The averaging problem in cosmology is perhaps the most important unsolved problem in mathematical cosmology. It has considerable importance for the correct interpretation of cosmological data. Cosmological observations [1–4], based on the assumption of a spatially homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) model plus small perturbations are usually interpreted as implying that there exists dark energy, the spatial geometry is flat, and that there is currently an accelerated expansion giving rise to the so-called ΛCDM-concordance model with $\Omega_m \sim 1/3$ and $\Omega_{de} \sim 2/3$. Although the concordance model is quite remarkable, it does not convincingly fit all data [3–6]. Unfortunately, if the underlying cosmological model is not a perturbation of an exact flat FLRW solution, the conventional data analysis and their interpretation is not necessarily valid.

The correct governing equations on cosmological scales are obtained by averaging the Einstein equations of general relativity. By assuming spatial homogeneity and isotropy on the largest scales, the inhomogeneities affect the dynamics though correction (backreaction) terms, which can lead to behaviour qualitatively and quantitatively different from the FLRW models. There are a number of theoretical approaches to the averaging problem. In the approach of Buchert [7] a cosmological space-time splitting is employed and only scalar quantities are averaged, whereby the Hamiltonian constraint and the Raychaudhuri equation can be replaced by their spatially averaged counterparts plus an integrability condition [7]. The Buchert approach is heuristic in that it is necessary to specify one more condition (not obtained from the field equations) in order to obtain a closed system of equations. The averaged Hamilton constraint can be written:

$$\Omega_m + \Omega_{\mathcal{R}} + \Omega_{\mathcal{Q}} + \Omega_{de} \ = \ 1 \ , \tag{1}$$

where $\Omega_m = \frac{8\pi G\langle\rho\rangle}{3H^2}$, $\Omega_{\mathcal{R}} = -\frac{\mathcal{R}}{3H^2}$, $\Omega_{\mathcal{Q}} = \frac{\mathcal{Q}}{3H^2}$, and Ω_{de} is the Hubble-normalized dark energy contribution (e.g., $\Omega_\Lambda \sim \frac{\Lambda}{6H^2}$, where Λ is the cosmological constant), and $H(t) = \dot{a}_D/a_D$ (where a_D is the averaged scale factor in some domain D). We define $\Omega_{tot} \equiv \Omega_m + \Omega_{de}$, so that $|\Omega_{tot} - 1|$ is given in terms of $\mathcal{Q}$, the kinematic variance (backreaction) term and $\mathcal{R}$, the averaged spatial curvature term (which is not necessarily isotropic).

The macroscopic gravity (MG) approach is an exact approach which gives a prescription for the correlation functions that emerge in an averaging of the Einstein field equations [8]. In [9] the MG equations were explicitly solved in a FLRW background geometry and it was found that the correlation tensor is of the form of a spatial curvature. This result was confirmed in subsequent work in which the spherically symmetric Einstein equations were explicitly averaged [10], and is consistent with the work of Buchert [7] (in the Newtonian limit and for exact spherically symmetric spaces [11]), with the results of averaging an exact Lemaître-Tolman-Bondi (LTB) spherically symmetric dust model [10], and with results from linear perturbation theory [12].

There is no question that the backreaction effect is real [13,14,7,9,10]. Spatial curvature must be taken into account in observational cosmology. The only question remaining is the potential significance of the resulting effect. However, even a small backreaction would be of importance; for example, a non-zero curvature, even at the 1 % level (i.e., $|\Omega_{\mathcal{R}}| \sim .01$), would have a significant effect on observations (for redshifts $z \geq 0.9$) [15]. This has been verified in [16], where values of $|\Omega_{\mathcal{R}}| \sim 0.05$ or larger were found to be consistent with observation (if one allows for a varying dark energy equation of state parameter).

The Wilkinson Microwave Anisotropy Probe (WMAP) has reported $\Omega_{tot} = 1.02 \pm 0.02$ [3]. In ΛCDM models type Ia supernovae data alone [1] prefers a slightly closed Universe [17]. Taken at face value this suggests $\Omega_{\mathcal{R}} = 0.02$. Models with non-negligible spatial curvature have been the subject of renewed interest recently [16]. There are other possible (non-dark energy) explanations for the SNIa data; indeed, it has been argued that models with no dark energy are consistent with supernova data and WMAP data (e.g., see [6] and references within). Combining these observational data with Large Scale Structure (LSS) observations such as the Baryon Acoustic Oscillations (BAO) data can put stringent limits on the curvature parameter in the context of adiabatic ΛCDM models [5]. However, these data analyses are very model- and prior-dependent [18], and the assumptions may be unjustified (or even inconsistent) and care is needed in the interpretation of data.

Using both the Buchert equations and cosmological perturbation theory [19], it has been found that $\mathcal{Q}$ is a pure second order term and $\mathcal{R}$ is of first order [14]. The exact results of [9,10] are consequently consistent with the results of perturbation theory and the results of Buchert [14,13] in the following sense. If $\mathcal{Q}$ is second order and $\mathcal{R}$ is first order, then we can solve the integrability condition to each order of approximation separately. To first order, if $\mathcal{Q} = 0$, then $\mathcal{R} \propto k_D/{a_D}^2$, and the averaged spatial curvature evolves like a constant–curvature model. Consequently, the exact results apply at first order. $\mathcal{R}$ is given in terms of the integration constant k_D, which depends on the averaging scale, the scale of inhomogeneities and the matter distribution density contrast. The dynamical effects are determined through $\Omega_{\mathcal{R}}$.

At second order we obtain effects that can be modelled by Buchert's heuristic approach, which can be thought of as arising from violations of the approximations and assumptions used in the exact approach. If $\mathcal{Q}$ is non-zero, in addition to the possible second order effects, $\mathcal{R}$ evolves differently to a_D^{-2} and this can, in turn, significantly affect cosmological observations [7]. Indeed, since the kinematic backreaction $\Omega_{\mathcal{Q}}$ decays more slowly than $\Omega_{\mathcal{R}}$ (and Ω_m), in the course of expansion the kinematic backreaction becomes dynamically more important and can become significant when the structure formation process "injects backreaction" (which could then be a possible explanation of the coincidence

problem) [13]. We may expect the largest deviations from the exact approach to arise from the fact that the Universe is not FLRW on a particular scale of observations. If L_s is the scale of averaging and L_F is an appropriate scale on which the Universe is FLRW, then $\mathcal{Q} \to 0$ as $L_s/L_F \to 1$. For small L_s/L_F, $\mathcal{Q} \sim (L_s/L_F)^2\mathcal{R}$. A typical scale $L_s \sim 50\ h^{-1}$ Mpc, a reasonable homogeneity scale is $L_h \gtrsim 200\ h^{-1}$ Mpc [20] and the Hubble radius $L_H \sim 3000\ h^{-1}$ Mpc. For these values, $(L_s/L_F)^2$ is typically in the range 0.01 to 0.1.

Let us consider the size of this affect in more detail. Robust estimators for intrinsic curvature fluctuations using realistically modelled clusters and voids in a Swiss-cheese model indicates that the dark energy effects can be reduced by up to about 30 % [7,21]. Hence the regional Friedmann curvature estimated on the regional Hubble scale can be large. A rough order of magnitude estimate for the variance implied by the observed density distribution of voids implies $|\Omega_\mathcal{Q}| \lesssim 0.2$ [13]. Since the value of the backreaction term $\mathcal{Q}$ depends on the velocity field inside the domain D, it has been suggested that peculiar–velocity catalogues may offer an alternative way of estimating $\mathcal{Q}$ [11]. The backreaction parameter has also been estimated in the framework of Newtonian cosmology [11]; it was found that the backreaction term can be quantitatively small in sufficiently large expanding domains of the Universe (e.g., $\Omega_\mathcal{Q} = 0.01$), but the dynamical influence of a non–vanishing backreaction on the other cosmological parameters can be large. Therefore, $\Omega_\mathcal{R} + \Omega_\mathcal{Q}$ can, in principal, be quite large.

This has suggested a possible scenario in which $\Omega_\mathcal{R} + \Omega_\mathcal{Q} \sim 0.2 - 0.5$. In this case it is possible for the observed acceleration to be explained (by backreaction) using the standard interpretation of observational data without resorting to dark energy. However, it is probably fair to say that this possible scenario is not supported by most researchers in the field [7,13].

There is a heuristic argument that $\Omega_\mathcal{R} \sim 10^{-2}$, which is consistent with CMB observations. Let us assume that $\Omega_\mathcal{R} \sim \epsilon$. We can estimate $\mathcal{Q} \sim \left\langle \sigma^2 \right\rangle_D$, where σ_D is the fluctuation amplitude; then $\Omega_\mathcal{Q} \sim \left\langle \sigma^2 \right\rangle_D / H_D^2 \sim \langle \delta \rangle_D \sim \epsilon^2$ (where δ is the density contrast) [14]. From CMB data we then have that $|\Omega_\mathcal{Q}| \sim 10^{-5}$ [22]. Consequently, we have that $|\Omega_\mathcal{R}| \lesssim 10^{-2}$. Indeed, from perturbation theory the dominant local corrections to redshift or luminosity distance go as $\nabla\Phi$, which is only suppressed as (L/L_H) (rather than go as the Newtonian potential Φ as naively expected, which is suppressed as $(L/L_H)^2$) [19]; for scales as large as a few hundred Mpc, the suppression is therefore mild, with $\Omega_\mathcal{Q} \sim 10^{-5}$ [13].

This suggests a second possible scenario in which $|\Omega_\mathcal{R}| \sim 0.01 - 0.02$. It must be appreciated that this value for $\Omega_\mathcal{R}$ is very large and will still have a significant dynamical effect. Note that $\Omega_\mathcal{R} >> \Omega_\gamma$, the energy density of radiation, and $\Omega_\mathcal{R} \sim \Omega_{lum}$, the energy density in luminous matter. Indeed, from nucleosynthesis bounds $\Omega_\mathcal{R}$ is comparable with

Ω_b, the total contribution of baryons to the normalized density.

There are two possibilities if $|\Omega_{\mathcal{R}}| \sim 0.01 - 0.02$. Perhaps cosmological data can be explained without dark energy through a small backreaction and a reinterpreation of cosmological observations; for example, many authors have studied observations in LTB models (albeit with contradictory conclusions), and the subject is currently under intense investigation [23]. Alternatively, $|\Omega_{\mathcal{R}}| \sim 0.01 - 0.02$, but dark energy is still needed for consistency with observations. However, a value of $|\Omega_{\mathcal{R}}| \sim 0.01 - 0.02$ would certainly necessitate a complete reinvestigation of cosmological observations. In addition, such a value cannot be naturally explained by inflation. From standard analysis, depending on the initial conditions and the details of a specific model of inflation, $|\Omega_{tot} - 1|$ would be extremely small. Indeed, any value for $|\Omega_{tot} - 1| >> 10^{-10}$ (say) would be very difficult to explain within the theory of inflation. Therefore, inflation indicates that spatial curvature is effectively zero, so that any non-zero residual curvature (e.g., at the one percent level) can only be naturally explained in terms of an averaging effect.

Clearly, averaging can have a very significant dynamical effect on the evolution of the Universe; the correction terms change the interpretation of observations so that they need to be accounted for carefully to determine if a model may be consistent with observations. Averaging may or may not explain the observed acceleration. However, it cannot be neglected, and a proper analysis of cosmological observations will not be possible without a comprehensive understanding of the affects of averaging. Indeed, any observation that is based on physics in the (weakly or strongly) nonlinear regime may well be influenced by the backreaction effect.

Let us discuss the cosmological observations further.

- The standard analysis of supernova type Ia data and CMB data in FLRW models cannot be applied directly when backreaction effects are present, because of the different behaviour of the spatial curvature [18]. Indeed, all data needs to be analysed within a particular inhomogeneous model, and not just an averaged version. Several studies of the LTB model have demonstrated that the effect of inhomogeneity on the luminosity distance can mimic acceleration [23].

- It is necessary to carefully identify observables actually measured by an experiment. For example, there are several different measures of the expansion rate and acceleration. In addition, there is the question of whether we are dealing with regional dynamics or global (averaged) dynamics in any particular observation. The determination of the Hubble parameter H_0 and the Hubble diagrams from supernovae type Ia (which need data at low redshift $z < 1$) are clearly based on local measurements. However, most observations of the CMB (e.g., the integrated Sachs-Wolfe effect, which can be probed at $z > 1$) and LSS (e.g., N-body simulations) are sensitive to

large-scale (averaged) properties of the Universe, rather than local ones. Indeed, integrated affects presumably depend not only on averaged values of parameters (e.g., $\mathcal{R}$) but also their actual values over an appropriate timescale.

- All of our deductions about cosmology from observations are based on light paths. The presence of inhomogeneities affects curved null geodesics [13,7] and can drastically alter observed distances when they are a sizable fraction of the curvature radius. The issue of light propagation in such an inhomogeneous and anisotropic spacetime should be studied in the context of a realistic quantitative model. In the real Universe, voids occupy a much larger region as compared to structures [24], hence for local observations light preferentially travels much more through underdense regions and the effects of inhomogeneities (voids and structures) on redshift and luminosity distance are likely to be significant.

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